Hard diffractive processes and non-perturbative matrix elements beyond leading twist: ρ_T-meson production

I. V. Anikin[1], *D.Yu. Ivanov*[2], *B. Pire*[3], *L. Szymanowski*[4], *S. Wallon*[5]
1. *Bogoliubov Laboratory of Theoretical Physics, JINR, 141980 Dubna, Russia*
2. *Sobolev Institute of Mathematics, 630090 Novossibirsk, Russia*
3. *CPHT, École Polytechnique, CNRS, 91128 Palaiseau Cedex, France*
4. *Soltan Institute for Nuclear Studies, PL-00-681 Warsaw, Poland*
5. *LPT, Université Paris-Sud, CNRS, 91405, Orsay, France* &
UPMC Univ. Paris 06, faculté de physique, 4 place Jussieu, 75252 Paris 05, France

1 Introduction

Studies of hard exclusive reactions rely on the factorization properties of the leading twist amplitudes [1] for deeply virtual Compton scattering and deep exclusive meson production. The leading twist distribution amplitude (DA) of a transversally polarized vector meson is chiral-odd, and hence decouples from hard amplitudes even when another chiral-odd quantity is involved [2] unless in reactions with more than two final hadrons [3]. Thus, transversally polarized $\rho-$meson production is generically governed by twist 3 contributions for which a pure collinear factorization fails due to the appearance of end-point singularities [4, 5]. The meson quark gluon structure within collinear factorization may be described by Distribution Amplitudes (DAs), classified in [6]. On the experimental side, in photo and electro-production and from moderate to very large energy [7, 8], the kinematical analysis of the final $\pi-$meson pair allows to measure the ρ_T-meson production amplitude, which is by no means negligible and needs to be understood in terms of QCD. Up to now, experimental information comes from electroproduction on a proton or nucleus. We will specifically concentrate on the case of very high energy collisions at colliders. Future progress in this range may come from real or virtual photon photon collisions [9, 10].

In the literature there are two approaches to the factorization of the scattering amplitudes in exclusive processes at leading and higher twists. The first approach [11, 5], which we will call Light-Cone Collinear Factorization (LCCF), is the generalization of the Ellis-Furmanski-Petronzio (EFP) method [12] to the exclusive processes, and deals with the factorization in the momentum space around the dominant light-cone direction. On the other hand, there exists a Covariant Collinear Factorization (CCF) approach in coordinate space succesfully applied in [6] for a systematic description of DAs of hadrons carrying different twists. Although being quite different and using different DAs, both approaches can be applied to the description of the

same processes. We have shown that these two descriptions are equivalent at twist 3 [13, 14]. We first establish a precise vocabulary between objects appearing in the two approaches. Then we calculate within both methods the impact factor $\gamma^* \to \rho_T$, up to twist 3 accuracy, and prove the full consistency between the two results. The key idea within LCCF is the invariance of the scattering amplitude under rotation of the light-cone vector n^μ (conjugated to the light-cone momentum of the partons), which we call n-independence condition. Combined with the equation of motions (EOMs), this reduces the number of relevant soft correlators to a minimal set. For ρ-production up to twist 3, this reduces a set of 7 DAs to 3 independent DAs which fully incodes the non-perturbative content of the ρ-wave function.

2 LCCF factorization of exclusive processes

2.1 Factorization beyond leading twist

The most general form of the amplitude for the hard exclusive process $A \to \rho\, B$ is, in the momentum representation and in axial gauge,

$$\mathcal{A} = \int d^4\ell \, \mathrm{tr}\Big[H(\ell)\,\Phi(\ell)\Big] + \int d^4\ell_1\, d^4\ell_2\, \mathrm{tr}\Big[H_\mu(\ell_1,\ell_2)\,\Phi^\mu(\ell_1,\ell_2)\Big] + \dots\,, \tag{1}$$

where H and H_μ are 2- and 3-parton coefficient functions, respectively. In (1), the soft parts are given by the Fourier-transformed 2- or 3-partons correlators which are matrix elements of non-local operators. To factorize the amplitude, we choose the dominant direction around which we decompose our relevant momenta and we Taylor expand the hard part. Let p and n be a large "plus" and a small "minus" light-cone vectors, respectively ($p \cdot n = 1$). Any vector ℓ is then expanded as

$$\ell_{i\,\mu} = y_i\, p_\mu + (\ell_i \cdot p)\, n_\mu + \ell^\perp_{i\,\mu}, \quad y_i = \ell_i \cdot n, \tag{2}$$

and the integration measure in (1) is replaced as $d^4\ell_i \longrightarrow d^4\ell_i\, dy_i\, \delta(y_i - \ell \cdot n)$. The hard part $H(\ell)$ is then expanded around the dominant "plus" direction:

$$H(\ell) = H(yp) + \left.\frac{\partial H(\ell)}{\partial \ell_\alpha}\right|_{\ell = yp} (\ell - y\,p)_\alpha + \dots \tag{3}$$

where $(\ell - y\,p)_\alpha \approx \ell^\perp_\alpha$ up to twist 3. To obtain a factorized amplitude, one performs an integration by parts to replace $\ell^\perp_\alpha$ by $\partial^\perp_\alpha$ acting on the soft correlator. This leads to new operators containing transverse derivatives, such as $\overline{\psi}\,\partial^\perp \psi$, thus requiring additional DAs $\Phi^\perp(l)$. Factorization in the Dirac space is then achieved by Fierz decomposition on a set of relevant Γ matrices. The amplitude is thus factorized as

$$\mathcal{A} \;=\; \int_0^1 dy\ \mathrm{tr}\left[H_{q\bar{q}}(y)\,\Gamma\right]\ \Phi^\Gamma_{q\bar{q}}(y) + \int_0^1 dy\ \mathrm{tr}\left[H^{\perp\mu}_{q\bar{q}}(y)\,\Gamma\right]\ \Phi^{\perp\Gamma}_{q\bar{q}\,\mu}(y)$$

$$+ \int_0^1 dy_1\, dy_2\, \mathrm{tr}\left[H^{\mu}_{q\bar{q}g}(y_1,y_2)\,\Gamma\right]\;\Phi^{\Gamma}_{q\bar{q}g\,\mu}(y_1,y_2)\,, \tag{4}$$

in which the first (second) line corresponds to the 2 (3)-parton contribution (see Fig.1). For ρ-meson production, the soft parts of $\mathcal{A}$ read, with $i\stackrel{\rightarrow}{D}_\mu = i\stackrel{\rightarrow}{\partial}_\mu + g\,A_\mu\,,$

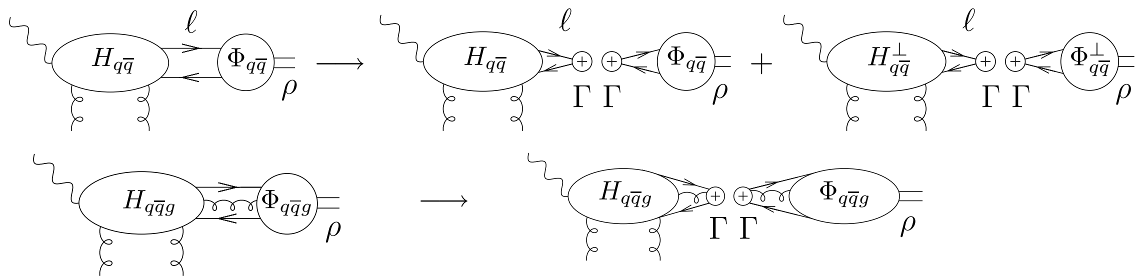


Figure 1: Factorization of 2- (up) and 3-parton (down) contributions in the example of the $\gamma^* \to \rho$ impact factor.

$$\Phi^{\Gamma}(y) \;=\; \int_{-\infty}^{+\infty} \frac{d\lambda}{2\pi}\, e^{-i\lambda y} \langle \rho(p)|\overline{\psi}(\lambda n)\,\Gamma\,\psi(0)|0\rangle \tag{5}$$

$$\Phi^{\Gamma}_{\rho}(y_1,y_2) \;=\; \int_{-\infty}^{+\infty} \frac{d\lambda_1 d\lambda_2}{4\pi^2}\, e^{-i\lambda y_1 \lambda_1 - i(y_2-y_1)\lambda_2} \langle \rho(p)|\overline{\psi}(\lambda_1 n)\,\Gamma\, i\stackrel{\longleftrightarrow}{D^T_\rho}(\lambda_2 n)\,\psi(0)|0\rangle\,.$$

2.2 Vacuum–to–rho-meson matrix elements up to twist 3

In the LCCF approach, the coordinates z_i in the parameterizations have to be fixed by the light-cone vector n. This is in contrast to the CCF approach where z lies on the light cone but does not corresponds to some fixed light-cone direction. The transverse polarization of the $\rho-$meson is defined by the conditions (at twist 3, $p_\rho \sim p$)

$$e_T \cdot n = e_T \cdot p = 0\,. \tag{6}$$

Keeping all the terms up to the twist-3 order with the axial (light-like) gauge, $n{\cdot}A = 0$, the matrix elements of quark-antiquark nonlocal operators can be written as (here, $z = \lambda n$ and $\stackrel{\mathcal{F}_1}{=}$ is the Fourier transformation with measure $\int_0^1 dy \exp\left[iy\, p\cdot z\right]$)

$$\langle \rho(p_\rho)|\overline{\psi}(z)\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}_1}{=} m_\rho\, f_\rho\left[\varphi_1(y)\,(e^*\cdot n)p_\mu + \varphi_3(y)\, e^*_{T\mu}\right], \tag{7}$$

$$\langle \rho(p_\rho)|\overline{\psi}(z)\gamma_5\gamma_\mu\psi(0)|0\rangle \stackrel{\mathcal{F}_1}{=} m_\rho\, f_\rho\, i\varphi_A(y)\,\varepsilon_{\mu\alpha\beta\delta}\, e^{*\alpha}_T p^\beta n^\delta\,, \tag{8}$$

where the corresponding flavour matrix has been omitted. The momentum fraction

y ($\overline{y}$) corresponds to the quark (antiquark). Denoting $\overleftrightarrow{\partial_\rho} = \frac{1}{2}(\overrightarrow{\partial_\rho} - \overleftarrow{\partial_\rho})$, the matrix elements of the quark-antiquark operators with transverse derivatives are

$$\langle \rho(p_\rho)|\overline{\psi}(z)\gamma_\mu i \overleftrightarrow{\partial_\alpha^T} \psi(0)|0\rangle \stackrel{\mathcal{F}_1}{=} m_\rho\, f_\rho\, \varphi_1^T(y)\, p_\mu e^*_{T\alpha} \tag{9}$$

$$\langle \rho(p_\rho)|\overline{\psi}(z)\gamma_5\gamma_\mu i \overleftrightarrow{\partial_\alpha^T} \psi(0)|0\rangle \stackrel{\mathcal{F}_1}{=} m_\rho\, f_\rho\, i\varphi_A^T(y)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e_T^{*\lambda} p^\beta\, n^\delta\,. \tag{10}$$

The matrix elements of quark-gluon nonlocal operators can be parameterized as[1]

$$\langle \rho(p_\rho)|\overline{\psi}(z_1)\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_{3\,\rho}^V\, B(y_1, y_2)\, p_\mu e^*_{T\alpha}, \tag{11}$$

$$\langle \rho(p_\rho)|\overline{\psi}(z_1)\gamma_5\gamma_\mu g A_\alpha^T(z_2)\psi(0)|0\rangle \stackrel{\mathcal{F}_2}{=} m_\rho\, f_{3\,\rho}^A\, iD(y_1, y_2)\, p_\mu\, \varepsilon_{\alpha\lambda\beta\delta}\, e_T^{*\,\lambda}\, p^\beta n^\delta\,. \tag{12}$$

Note that φ_1 corresponds to the twist-2, and B and D to the genuine (dynamical) twist-3, while functions $\varphi_3,\ \varphi_A, \varphi_1^T,\ \varphi_A^T$ contain both parts: kinematical (à la Wandzura-Wilczek, noted WW) twist-3 and genuine (dynamical) twist-3.

We now recall and rewrite the original CCF parametrizations of the ρ DAs [6], adapting them to our case when vector meson is produced in the final state, and limiting ourselves to the twist 3 case. The formula for the axial-vector correlator is

$$\langle \rho(p_\rho)|\overline{\psi}(z)\,[z,\,0]\,\gamma_\mu\gamma_5\psi(0)|0\rangle = \frac{1}{4} f_\rho\, m_\rho\, \varepsilon_\mu{}^{e_T^*\,p\,z} \int\limits_0^1 dy\, e^{iy(p\cdot z)}\, g_\perp^{(a)}(y)\,, \tag{13}$$

where we denote $\varepsilon_\mu{}^{e_T^*\,p\,z} = \varepsilon_\mu{}^{\alpha\beta\gamma} e^*_{T\alpha}\, p_\beta\, z_\gamma$, and in which enters the Wilson line

$$[z_1,\, z_2] = P\exp\left[ig\int\limits_0^1 dt\,(z_1 - z_2)_\mu A^\mu(t\,z_1 + (1-t)\,z_2\right]\,. \tag{14}$$

The transverse vector e_T is orthogonal to the light-cone vectors p and z, and reads

$$e_{T\mu} = e_\mu - p_\mu \frac{e\cdot z}{p\cdot z} - z_\mu \frac{e\cdot p}{p\cdot z}\,. \tag{15}$$

Thus in the CCF parametrization the notion of "transverse" is different with respect to the one of LCCF defined by Eq.(6): as we discuss later in sec.3 the coordinate z on the light-cone and the light-cone vector n point in two different directions. It is thus useful to rewrite the original CCF parametrization in terms of the full meson polarization vector e. This is already done for the axial-vector correlator (13) since

[1]The symbol $\stackrel{\mathcal{F}_2}{=}$ means $\int\limits_0^1 dy_1 \int\limits_0^1 dy_2 \exp\left[iy_1\, p\cdot z_1 + i(y_2 - y_1)\, p\cdot z_2\right]$.

due to the properties of fully antisymmetric tensor $\epsilon_{\mu\nu\alpha\beta}$ one can use e instead of e_T in the r.h.s. of (13). The definition of 2-parton vector correlator of a ρ-meson reads

$$\langle\rho(p_\rho)|\overline{\psi}(z)\,[z,\,0]\,\gamma_\mu\psi(0)|0\rangle = f_\rho\, m_\rho \int\limits_0^1 \, dy\, e^{iy(p\cdot z)} \left[p_\mu\, \frac{e^*\cdot z}{p\cdot z}\phi_\parallel(y) + e^*_{T\mu}\, g_\perp^{(v)}(y)\right]\,, \quad (16)$$

which can be rewritten after integration by parts in a form which only involve e,

$$\langle\rho(p_\rho)|\overline{\psi}(z)\,[z,\,0]\,\gamma_\mu\psi(0)|0\rangle = f_\rho\, m_\rho \int\limits_0^1 dy\, e^{iy(p\cdot z)} \left[-i\, p_\mu\,(e^*\cdot z)\, h(y) + e^*_\mu\, g_\perp^{(v)}(y)\right]\,, \quad (17)$$

with $h(y) = \int\limits_0^y dv\, \left(\phi_\parallel(v) - g_\perp^{(v)}(v)\right)$ and $\overline{h}(y) = \int\limits_0^y dv\, \left(g_3(v) - g_\perp^{(v)}(v)\right)$.

For quark-antiquark-gluon correlators the parametrizations of Ref.[6] have the forms

$$\begin{aligned}
&\langle\rho(p_\rho)|\overline{\psi}(z)[z,t\,z]\gamma_\alpha g\, G_{\mu\nu}(t\,z)[t\,z,0]\psi(0)|0\rangle \\
&= \; -ip_\alpha[p_\mu e^*_{\perp\nu} - p_\nu e^*_{\perp\mu}]m_\rho\, f^V_{3\,\rho} \int D\alpha\, V(\alpha_1,\alpha_2)\, e^{i(p\cdot z)(\alpha_1 + t\,\alpha_g)}\,, \qquad (18)\\
&\langle\rho(p_\rho)|\overline{\psi}(z)[z,t\,z]\gamma_\alpha\gamma_5 g\, \tilde{G}_{\mu\nu}(t\,z)[t\,z,0]\psi(0)|0\rangle \\
&= \; -p_\alpha[p_\mu e^*_{\perp\nu} - p_\nu e^*_{\perp\mu}]m_\rho\, f^A_{3\,\rho} \int D\alpha\, A(\alpha_1,\alpha_2)\, e^{i(p\cdot z)(\alpha_1 + t\,\alpha_g)}\,, \qquad (19)
\end{aligned}$$

where α_1, α_2, α_g are momentum fractions of quark, antiquark and gluon respectively inside the $\rho-$meson, $\int D\alpha = \int\limits_0^1 d\alpha_1 \int\limits_0^1 d\alpha_2 \int\limits_0^1 d\alpha_g\, \delta(1-\alpha_1-\alpha_2-\alpha_g)$ and $\tilde{G}_{\mu\nu} = -\frac{1}{2}\epsilon_{\mu\nu\alpha\beta}G^{\alpha\beta}$. In the axial gauge $A\cdot n = 0$, $n^2 = 0$, the 3-parton correlators thus reads

$$\langle\rho(p_\rho)|\overline{\psi}(z)\gamma_\mu g A_\alpha(tz)\psi(0)|0\rangle = -p_\mu\, e^*_{T\alpha} m_\rho\, f^V_{3\rho} \int \frac{D\alpha}{\alpha_g}\, e^{i(p\cdot z)(\alpha_1 + t\,\alpha_g)} V(\alpha_1,\alpha_2)\,, \qquad (20)$$

$$\langle\rho(p_\rho)|\overline{\psi}(z)\gamma_\mu\gamma_5 g A_\alpha(tz)\psi(0)|0\rangle = -ip_\mu \frac{\varepsilon_\alpha^{\;z\,p\,e^*_T}}{(p\cdot z)} m_\rho\, f^A_{3\rho} \int \frac{D\alpha}{\alpha_g}\, e^{i(p\cdot z)(\alpha_1 + t\alpha_g)} A(\alpha_1,\alpha_2)\,. \quad (21)$$

2.3 Minimal set of DAs and dictionary

The correlators introduced above are not independent. First, they are constrained by the QCD EOMs for the field operators entering them (see, for example, [5]). In the simplest case of fermionic fields, they follow from the vanishing of matrix elements $\langle (i\hat{D}(0)\psi(0))_\alpha\, \overline{\psi}_\beta(z)\rangle = 0$ and $\langle \psi_\alpha(0)\, i(\hat{D}(z)\overline{\psi}(z))_\beta\rangle = 0$ due to the Dirac equation,

then projected on different Fierz structure. They read, with $\zeta_3^{V(A)} = \frac{f_{3\rho}^{V(A)}}{f_\rho}$,

$$\overline{y}_1\,\varphi_3(y_1) + \overline{y}_1\,\varphi_A(y_1) + \varphi_1^T(y_1) + \varphi_A^T(y_1) = -\int_0^1 dy_2\,\left[\zeta_3^V\,B(y_1,\,y_2) + \zeta_3^A\,D(y_1,\,y_2)\right],\ (22)$$

$$y_1\,\varphi_3(y_1) - y_1\,\varphi_A(y_1) - \varphi_1^T(y_1) + \varphi_A^T(y_1) = -\int_0^1 dy_2\,\left[-\zeta_3^V\,B(y_2,\,y_1) + \zeta_3^A\,D(y_2,\,y_1)\right].(23)$$

Second, contrarily to the light-cone vector p related to the out-going meson momentum, the second light-cone vector n (with $p \cdot n = 1$), required for the parametrization of the needed LCCF correlators, is arbitrary, and the scattering amplitudes should be $n-$independent. This condition expressed at the level of the *full amplitude* of any process can be reduced to a set of conditions involving only the soft correlators, and thus the DAs. For processes involving ρ_T production up to twist 3 level, we obtained

$$\frac{d}{dy_1}\varphi_1^T(y_1) + \varphi_1(y_1) - \varphi_3(y_1) + \zeta_3^V \int_0^1 \frac{dy_2}{y_2 - y_1}\,(B(y_1, y_2) + B(y_2, y_1)) = 0\,, \quad (24)$$

$$\frac{d}{dy_1}\varphi_A^T(y_1) - \varphi_A(y_1) + \zeta_3^A \int_0^1 \frac{dy_2}{y_2 - y_1}\,(D(y_1, y_2) + D(y_2, y_1)) = 0\,. \quad (25)$$

The starting point is to exhibit the n-dependency of the polarization vector for transverse ρ which enters in the parametrization of twist 3 correlators, which is

$$e^{*T}_\mu = e^*_\mu - p_\mu\, e^* \cdot n\,. \quad (26)$$

The $n-$independence condition of the amplitude $\mathcal{A}$ can thus be written as

$$\frac{d\mathcal{A}}{dn^\mu} = 0\,, \qquad \text{where} \quad \frac{d}{dn^\mu} = \frac{\partial}{\partial n^\mu} + e^*_\mu \frac{\partial}{\partial(e^* \cdot n)}\,, \quad (27)$$

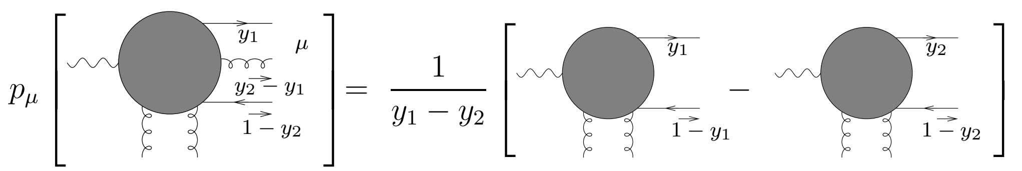
where e denotes now both longitudinal and transverse polarizations. This will lead to Eqs.(24, 25) on the DAs. Although Eqs.(24, 25) were derived explicitly in [14] using as a tool the explicit example of the $\gamma^* \to \rho$ impact factor, this proof is independent of the specific process under consideration, and only rely on general arguments based on Ward identities. For the $\gamma^* \to \rho$ impact factor, one needs to consider 2-parton contributions both without (see Fig.2) and with (see Fig.3) transverse derivative, as well as 3-parton contributions (see Figs.4, 5). The equations (24, 25) are obtained by considering the consequence of the $n-$independency on the contribution to the C_F color structure[2]. To illustrate the idea which is behind this proof, let us consider

[2]The $n-$independency condition applied to the N_c structure is automatically satisfied [14].

Eq.(24), which corresponds to the vector correlator contributions with C_F invariant. In the case of the 3-parton vector correlator (11), due to (26) the dependency on n enters linearly and only through the scalar product $e^* \cdot n$. Thus, the action on the amplitude of the derivative d/dn defined in (27) can be extracted by the replacement $e^*_\alpha \to -p_\alpha$, which means in practice that the Feynman rule entering the coupling of the gluon inside the hard part should be replaced by $-g\, t^a\, \gamma^\alpha\, p_\alpha$. Then, using the Ward identity for the hard part, it reads

$$(y_1 - y_2)\mathrm{tr}\left[H^\rho_{q\bar{q}g}(y_1, y_2)\, p_\rho\, \not{p}\right] = \mathrm{tr}\left[H_{q\bar{q}}(y_1)\, \not{p}\right] - \mathrm{tr}\left[H_{q\bar{q}}(y_2)\, \not{p}\right] ,$$

which can be seen graphically as



(28)

as is shown in details in [14]. Eq.(28) implies that the 3-parton contribution to the $n-$independency condition can be expressed as the convolution of a 2-parton hard part with the last term of the l.h.s of Eq.(24). A similar treatment can be applied to the 2-parton correlators with transverse derivative whose contributions can be viewed as 3-parton processes with vanishing gluon momentum. This leads to the convolution of the first term of the l.h.s of Eq.(24) with the *same* 2-parton hard part appearing after applying Ward identities to the 3-partons contributions. The second term, with φ_1, of the l.h.s of Eq.(24) originates from the 2-parton vector correlator and corresponds to the contribution for the longitudinally polarized ρ with $e_L \sim p$. The third term with φ_3 corresponds to the contribution of the same correlator for the polarization vector of ρ_T written as in Eq.(26). To finally get Eq.(24), we used the fact that each individual term obtained above when expressing the $n-$independency condition involve the *same* 2-parton hard part, convoluted with the Eq.(24) through an integration over y_1 . The arguments used above, based on the collinear Ward identity, are clearly independent of the detailled structure of this resulting 2-parton hard part, implying that Eq.(24) itself should be satisfied. A similar treatment for axial correlators leads to Eq.(25), as we have shown in [14].

Solving the 4 equations $(22, 23,\ 24, 25)$ now reduces the set of 7 DAs to the set of the 3 independent DAs φ_1 (twist 2) and B, D (genuine twist 3). We write $\varphi_3(y)$, $\varphi_A(y)$, $\varphi_1^T(y)$ and $\varphi_A^T(y)$ generically denoted as $\varphi(y)$ as $\varphi(y) = \varphi^{WW}(y) + \varphi^{gen}(y)$ where $\varphi^{WW}(y)$ and $\varphi^{gen}(y)$ are WW and genuine twist-3 contributions, respectively. The WW DAs are solutions of Eqs. (22, 23, 24, 25) with vanishing B, D and read

$$\varphi^{T\, WW}_{A(1)}(y_1) = \frac{1}{2}\left[-\bar{y}_1 \int\limits_0^{y_1} \frac{dv}{\bar{v}}\varphi_1(v) - (+)\, y_1 \int\limits_{y_1}^{1} \frac{dv}{v}\varphi_1(v)\right] , \tag{29}$$

The solution of the set of equations for the genuine twist-3 φ^{gen} is given in Ref.[14].

The dictionary betweeen the 3-parton DAs in LCCF and CCF approaches is

$$
\begin{aligned}
B(y_1,\, y_2) &= -\frac{V(y_1,\, 1-y_2)}{y_2-y_1}\,,\ D(y_1,\, y_2) = -\frac{A(y_1,\, 1-y_2,\, y_2-y_1)}{y_2-y_1}\,, \\
\varphi_1(y) &= \phi_{\|}(y)\,, \quad \varphi_3(y) = g_{\perp}^{(v)}(y)\,, \quad \varphi_A(y) = -\frac{1}{4}\frac{\partial g_{\perp}^{(a)}(y)}{\partial y}\,.
\end{aligned} \tag{30}
$$

3 $\gamma^* \to \rho_T$ impact factor up to twist three accuracy

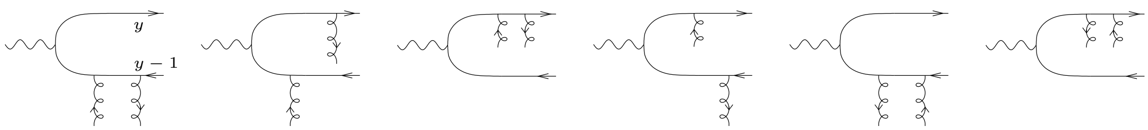


Figure 2: The 6 hard diagrams attached to the 2-parton correlators, which contribute to the $\gamma^* \to \rho$ impact factor, with momentum flux of external line along p_1 direction.

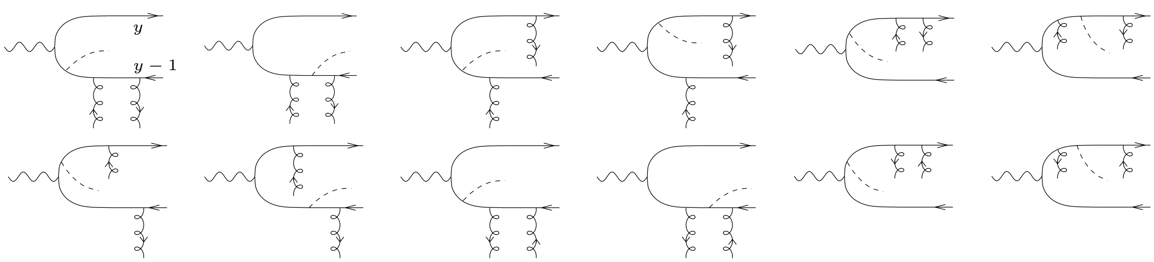


Figure 3: The 12 contributions arising from the first derivative of the 6 hard diagrams attached to the 2-parton correlators, which contribute to the $\gamma^* \to \rho$ impact factor.

The $\gamma^* \to \rho$ impact factor enters the description of high energy reactions in the k_T-factorization approach, e.g. $\gamma^*(q) + N \to \rho_T(p_1) + N$ or

$$
\gamma^*(q) + \gamma^*(q') \to \rho_T(p_1) + \rho(p_2) \tag{31}
$$

where the virtual photons carry large squared momenta $q^2 = -Q^2$ $(q'^2 = -Q'^2)$ $\gg \Lambda^2_{QCD}$, and the Mandelstam variable s obeys the condition $s \gg Q^2$, Q'^2, $-t \simeq \underline{r}^2$. The hard scale which justifies the applicability of perturbative QCD is set by Q^2 and Q'^2 and/or by t. Neglecting meson masses, one considers for reaction (31) the light cone vectors p_1 and p_2 as the vector meson momenta $(2\,p_1 \cdot p_2 = s)$. In this Sudakov basis (transverse euclidian momenta are denoted with underlined letters), the impact

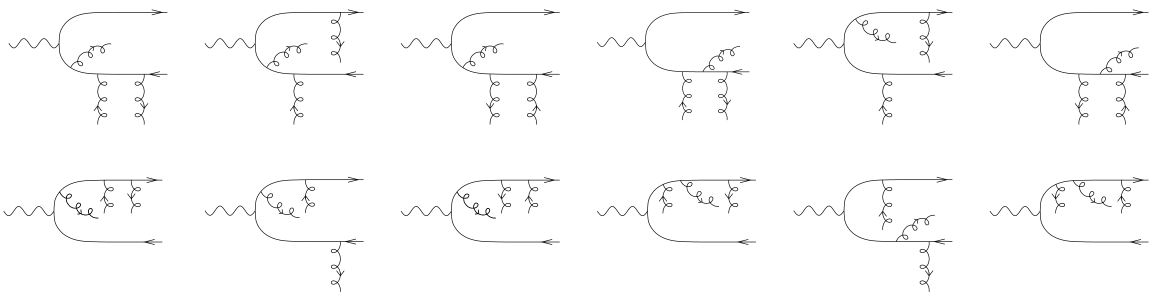


Figure 4: The 12 "Abelian" type contributions from the hard scattering amplitude attached to the 3-parton correlators for the $\gamma^* \to \rho$ impact factor.

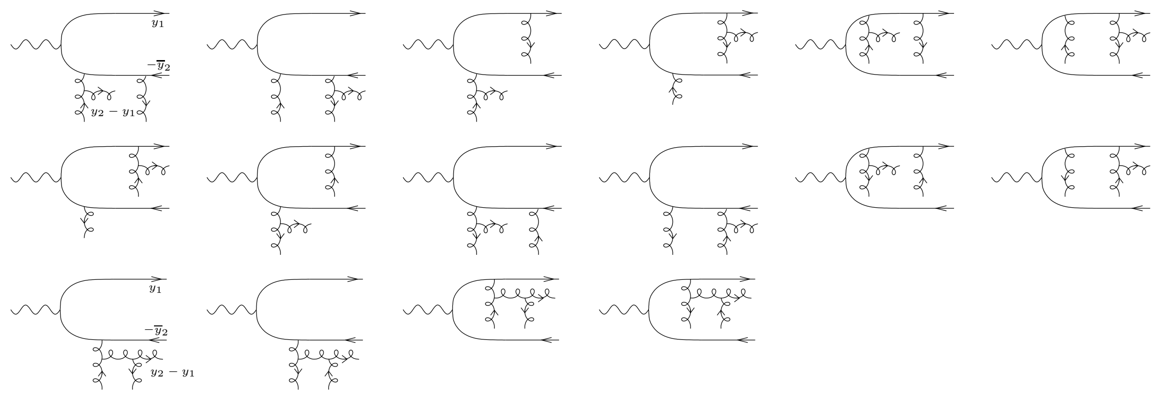


Figure 5: The 16 "non-abelian" (up: one triple gluon vertex, down: two triple gluon vertices) contributions to the $\gamma^* \to \rho$ impact factor.

representation of the scattering amplitude for the reaction (31) is

$$\mathcal{M} = \frac{is}{(2\pi)^2} \int \frac{d^2\underline{k}}{\underline{k}^2} \Phi_1^{ab}(\underline{k},\, \underline{r} - \underline{k}) \int \frac{d^2\underline{k}'}{\underline{k}'^2} \Phi_2^{ab}(-\underline{k}',\, -\underline{r} + \underline{k}') \int\limits_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{s}{s_0}\right)^{\omega} G_\omega(\underline{k}, \underline{k}', \underline{r})\,. \tag{32}$$

We focus here on the impact factor $\Phi^{\gamma^* \to \rho}$ of the subprocess[3] $g(k_1, \varepsilon_1) + \gamma^*(q) \to g(k_2, \varepsilon_2) + \rho_T(p_1)$. It is the integral of the S-matrix element $\mathcal{S}_\mu^{\gamma_T^*\, g \to \rho_T\, g}$ with respect to the Sudakov component of the t-channel k momentum along p_2 , or equivalently the integral of its κ-channel discontinuity ($\kappa = (q + k_1)^2$)

$$\Phi^{\gamma^* \to \rho}(\underline{k},\, \underline{r} - \underline{k}) = e^{\gamma^* \mu} \, \frac{1}{2s} \int\limits_0^{+\infty} \frac{d\kappa}{2\pi} \, \mathrm{Disc}_\kappa \, \mathcal{S}_\mu^{\gamma^*\, g \to \rho\, g}(\underline{k},\, \underline{r} - \underline{k})\,. \tag{33}$$

Note that within k_T-factorization, the description of impact factor for produced hadron described within QCD collinear approach requires a modification of twist

[3]The two reggeized gluons have so-called non-sense polarizations $\varepsilon_1 = \varepsilon_2^* = p_2\sqrt{2/s}$.

counting due to the off-shellness of the $t-$channel partons. Therefore, when here we say "up to twist 3" we only mean twist counting from the point of view of the collinear factorization of the produced $\rho-$meson, and not of the whole amplitude, e.g. $\gamma^*\, p \to \rho\, p$ or $\gamma^*\, \gamma^* \to \rho\, \rho$. We now consider the forward limit for simplicity. In order to describe the collinear factorization of ρ-production inside the impact factor (33), we note that the kinematics of the general approach discussed in section 2 is related to our present kinematics for the impact factor (33) by setting $p = p_1$, while a natural choice for n is obtained by setting $n = p_2/(p_1 \cdot p_2)$

We now compare the LCCF and CCF approaches, and show that they give identical results, when using the dictionary (30). The calculation of the $\gamma^*_L \to \rho_L$ impact factor is standard [15]. Within LCCF, it receives contribution only from the diagrams with quark-antiquark correlators, and it is given by contributions from the p_μ term of the correlators (7) of twist 2. It involves the computation of the 6 diagrams of Fig.2. We now consider the $\gamma^*_T \to \rho_T$ transition. The 2-parton contribution contains the terms arising from the diagrams of Fig.2, where the quark-antiquark correlators have no transverse derivative, and from the diagrams of Fig.3, where the quark-antiquark correlators stand with a transverse derivatives (denoted with dashed lines). The contributions of 3-parton correlators are of two types, the first one being of "abelian" type (see Fig.4) and the second involving non-abelian couplings (see Fig.5). The full result can be decomposed into spin-non-flip and spin-flip parts, respectively proportional to $T_{n.f.} = -(e_\gamma \cdot e^*_T)$, and $T_{f.} = \frac{(e_\gamma \cdot k_\perp)(e^*_T \cdot k_\perp)}{\underline{k}^2} + \frac{(e_\gamma \cdot e^*_T)}{2}$, and reads

$$\Phi^{\gamma^*_T \to \rho_T}(\underline{k}^2) = \Phi^{\gamma^*_T \to \rho_T}_{n.f.}(\underline{k}^2)\, T_{n.f.} + \Phi^{\gamma^*_T \to \rho_T}_{f.}(\underline{k}^2)\, T_f\,, \tag{34}$$

whose lenghty expressions are given in Ref.[14]. The gauge invariance of the considered impact factor is checked by the vanishing of our results for $\Phi_{f.}$ and $\Phi_{n.f.}$ when $\underline{k}^2 = 0$. The vanishing of the "abelian", i.e. proportional to C_F part of $\Phi_{n.f.}$ is particularly subtle since it appears as a consequence of EOMs (22, 23). Thus, the expression for the $\gamma^* \to \rho_T$ impact factor has finally a gauge-invariant form only provided the genuine twist 3 contributions have been taken into account. Finally, we note that end-point singularities do not occur here, both in WW approximation and in the full twist-3 order approximation, due to the $\underline{k}$ regulator specific of k_T-factorisation[4].

We now calculate the impact factor using the CCF parametrization of Ref.[6] for vector meson DAs. We need to express the impact factor in terms of hard coefficient functions and soft parts parametrized by light-cone matrix elements. The standard technique here is an operator product expansion on the light cone, $z^2 \to 0$, which naturally gives the leading term in the power counting and leads to the described above factorized structure. Unfortunately we do not have an operator definition for an impact factor, and therefore, we have to rely in our actual calculation on the perturbation theory. However the $z^2 \to 0$ limit of any single diagram is given in

[4] This does not preclude the solution of the well known end-point singularity problem [4, 16].

terms of light-cone matrix elements without any Wilson line insertion between the quark and gluon operators, like $\langle V(p_V)|\overline{\psi}(z)\gamma_\mu\psi(0)|0\rangle$ (we call them as perturbative correlators). Actually we need to combine together contributions of quark-antiquark and quark-antiquark gluon diagrams in order to obtain a final gauge invariant result. At twist 3 level, expanding the Wilson line at first order, one can show that

$$\langle\rho(p_\rho)|\overline{\psi}(z)\gamma_\mu\psi(0)|0\rangle|_{z^2\to 0} = \\ f_\rho\, m_\rho \left[-i\, p_\mu(e^*\cdot z)\int\limits_0^1 dy\, e^{iy(p\cdot z)}(h(y)-\tilde{h}(y)) + e^*_\mu \int\limits_0^1 dy\, e^{iy(p\cdot z)} g_\perp^{(v)}(z)\right], \quad (35)$$

where $\tilde{h}(y) = \zeta_3^V \int\limits_0^y d\alpha_1 \int\limits_0^{\bar{y}} d\alpha_2 \frac{V(\alpha_1,\alpha_2)}{\alpha_g^2}$, with an analogous result for the axial-vector correlator. Comparing the obtained result (35) for the perturbative correlators with initial parameterizations (17) we see that at twist 3-level the net effect of the Wilson line is just some renormalization of the h function in the case of vector correlator. For the axial-vector we obtain in addition to the function $g_\perp^a$ renormalization a new Lorentz structure (which does not contribute to the impact factor).

Based on the dictionary (30) and on the solution of Eqs.(22, 23, 24, 25), we got an exact equivalence between our two LCCF and CCF results, as proven in Ref.[14].

4 Conclusion

We compare the momentum space LCCF and the coordinate space CCF methods, illustrated here for ρ-meson production up to twist 3 accuracy. The crucial point is the use of Lorentz invariance constraints formulated as the n-independence of the scattering amplitude within LCCF method, which leads to the necessity of taking into account the contribution of 3-parton correlators. Our results for the $\gamma^*\rho$ impact factor in both methods are equivalent, based on our dictionary.

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